TECHNOLOGY
TANDEM

INCLUDES ANSWERS

GSDMC
HIGH SCHOOL
MATH FIELD DAY

MARCH 10, 2007

Sponsored by:
CUYAMACA COMMUNITY COLLEGE

INSTRUCTIONS:

You may work in pairs and field a maximum of two teams per school.

We will supply each team member with a TI-84 Graphing Calculator. Please leave your personal calculators at the front of the room.

Work as many of the problems as you can in the time allotted. You should not expect to finish every problem, so choose the ones you can readily complete.

The number of points earned will determine the winners. Be aware there is **NO penalty for guessing**.

The judges must score some of the problems on the spot. In this case, one team member will take the calculator to the appropriate judge for immediate grading. The judge will note the score for that question and will add it to your team score after the competition is over.

In order to improve this event for next year's competition, we would appreciate your feedback. Thanks for participating!

You may keep this test to prepare for next year's Technology Tandem competition, as the contents of the event may be similar to this year's event.

Each answer is worth five points.

1. Find the prime factorization of 43358976. (2^8)(3^5)(17)(41)

2. Find the prime factorization of 2736832813. (11^3)(13^2)(23^3)

3. Find the prime factorization of 90874860. (2^2)(5)(17)(41)(53)(123)

4. Find the smallest pair of twin primes greater than 40. 41 & 43

5. Find the smallest pair of twin primes greater than 2007. 2027 & 2029

6. Find the next pair of twin primes greater than 2007 (i.e. the first pair greater than the pair found in the previous problem). 2081 & 2083

7. Find the first pair of twin primes greater than one million.
1 000 037 & 1 000 039

8. Find the prime factorization of 70637. 7*10091

9. Find a pair of twin primes between 8000 and 8100. 8009 & 8011

10. Find the greatest common factor of 384870629 and 1110439. 18821

11. Find the least common multiple of 3145149 and 22818509. 5544897687

12. How many real solutions does $\frac{500 \cdot \sin(x)}{x} = 0.05 \cdot x^2 - 5$ have? 14

13. Solve $\sqrt{\dfrac{\dfrac{-(x+1)!+(x+2)!+(x-3)!-x!}{(x-4)!(x-1)!}}{x!+3\cdot 5\cdot 7}+8002-36163=2007}$ x = 21

14. Solve $\left(x^3-6x^2-10x-2039\right)^{\frac{x^2-18x-24}{\sqrt{x^4-3x^3+13x^2+7x+756}}}=191$. x = 36

15. Let k be the least positive integer that is divisible by each of the first ten natural numbers. Find $\left(\dfrac{k}{5005}+144\right)^{\frac{1}{6}}$. 6

16. What is the exact sum of all the intercepts of the graph of $f(x)=2x^2+5x-25$? -148/5

17. Find the exact value of $\sum_{k=1}^{\infty}\dfrac{k}{4^k}$. 4/9

18. How many times does $y=20\cos(200x)$ take on the value 15 on the interval $[0,1]$? 64

19. $\sqrt{2\sqrt{2\sqrt{2\ldots}}}=$ 2

20. How many pairs of integers (*a*, *b*) are there for which $a^2-b^2=256$. 14

21. Create a graph in polar coordinates of a flower with ten petals where each petal is 3 units long and a tip of a petal intersects with each axis. Show the graph to an instructor and be sure to set the graphing window and axes scales so that the instructor can readily see the lengths of the petals, the number of petals, and their placement with regard to the axes (bring your answer sheet with you).

AREA OF A TRIANGLE:

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) can be determined by evaluating the following determinant:

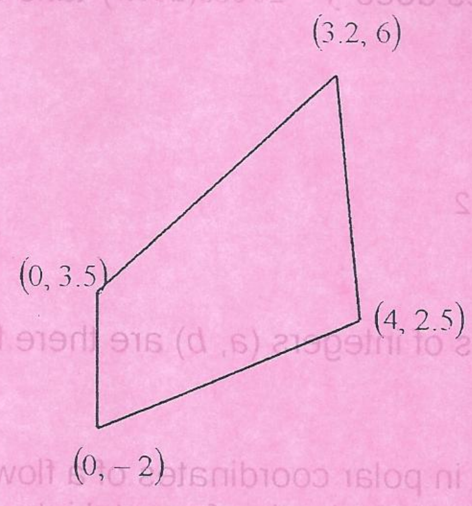
$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Where the $(\pm)$ symbol indicates that the appropriate sign should be chosen to yield a positive area. Find the areas of the following triangles, determined by the given points. Round answers to the nearest tenth if necessary.

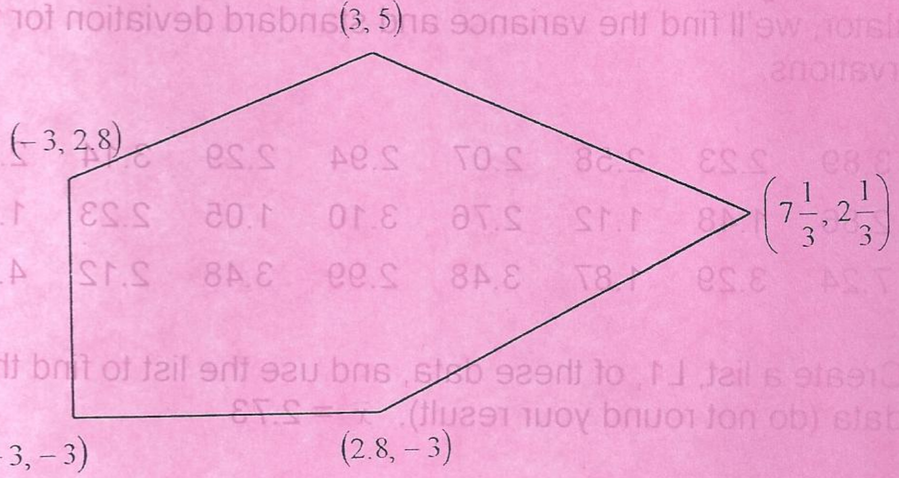
22. Triangle's vertices: $(1, 3)$, $(2, 7)$, and $(8, 4)$. 13.5

23. Triangle's vertices: $(-5, 4)$, $(e^2, 4)$, and $\left(6, \frac{-\sqrt{37}}{2}\right)$. 43.6

24. Find the area of the region defined be the vertices given below (note: the shape is not drawn to scale). 17.6



25. Find the area of the region defined be the vertices given below (note: the shape is not drawn to scale). 58.2



26. Find all values of y for which the triangle with the following vertices has an area of 100: $(-1, 5)$, $(-6, -2)$, $(4, y)$. y = -28, 52

STATISTICS:

Without using the variance and standard deviation functions of your graphing calculator, we'll find the variance and standard deviation for the following data observations.

3.89	2.23	2.58	2.07	2.94	2.29	3.14	2.15	1.95	0.51
2.86	1.48	1.12	2.76	3.10	1.05	2.23	1.70	5.57	2.81
7.24	3.29	1.87	3.48	2.99	3.48	2.12	4.69	2.29	2.02

27. Create a list, L1, of these data, and use the list to find the mean, $\bar{x}$, for these data (do not round your result). $\bar{x}$ = 2.73

28. Create a second list, L2, of the deviations from the mean; i.e. calculate $x-\bar{x}$, for each observation, x, in L1 and store these deviations in L2. Sum L2. (Do not round your result). 0

29. Create a third list, L3, of the squares of the deviations in L2; i.e. calculate $(x-\bar{x})^2$ for each observation, x. To find the variance for the original data observations, find the arithmetic mean of the entries in L3. Note: our variance may differ from applying the graphing calculator's variance function to the original data observations. (Do not round your result). Var = 1.74482

30. The standard deviation, sd, for the original data observations is the square root of the variance found in the previous problem. Find the standard deviation, sd (round to the nearest thousandth if necessary). sd = 1.321

31. The z-score (or standard score) for any given observation, x, is $\frac{x-\bar{x}}{sd}$.

Create a fourth list, L4, of the z-scores for the original data observations. What is the maximum z-score in the list (rounded to the nearest thousandth, if necessary). 3.414

(# 32 – 35) In a seven-year study of ten of the nation's most selective colleges and universities, researchers found that a student applicant has a better chance of being accepted to a college through early decision rather than by regular decision. The table below shows a comparison of SAT scores (out of 1600) and acceptance rates by both systems.

Let $E(s)$ and $R(s)$ be the percentages of early decision and regular decision applicants, respectively, which were accepted.

SAT Score	% Accepted Early	% Accepted Regular
1145	21	10
1245	35	17
1345	52	31
1445	70	48
1550	93	72

32. Find the regression equations for E and R. (For each model – be sure to determine which would be the best fit –**either linear or exponential**). Round the constants to 4 decimal places. $E(s)$ = 0.1774x – 184.5230 $R(s)$ = 0.0370*(1.0049)^x

33. For early decision applicants who score 1425 points, what percentage get accepted? What about for regular decision applicants who score 1425 points? (Round to the nearest hundredth). $E(1425)$ = 68.27% $R(1425)$ = 30.20%

34. For what score do half of early decision applicants get accepted? What about for regular decision applicants? (Round to the nearest whole number). Early: 1322 Regular: 1475

35. Use your graphing calculator to find the intersection point(s) of the graphs of E and R. What does this point(s) represent in terms of the situation?

(1081, 7.31) 7.31% of all applicants are accepted with an SAT score of 1081.
(1624, 103.52) 103.52% of all applicants is impossible as is an SAT score above 1600.

ALGEBRA TOPICS:

Unless you are directed to do otherwise, round irrational numbers to the nearest hundredth. Do not round rational numbers.

36. Use your calculator to find the equation of the quadratic function that passes through the following points (do not round):

(-2, 21.4), (3, 5.4), and (10, 25) .5x^2 – 3.7x + 12

37. Use your calculator to find the equation of the cubic function that passes through the following points (do not round): 3.78543x^3 – 5.21429x + 24.01

(-5, -423.0973), (-2, 4.15514), (5, 471.1173), and (10, 3757.2971)

38. Use your calculator to find the equation of the logarithmic function that passes through the following points:

$\left(e^{-3}, 12.46\right), \left(e^{2}, 0.06\right), \left(e^{5}, -7.38\right)$ $5.02 - 2.48\ln(x)$

39. Use your calculator to find the equation of the exponential function passing through the points:

(2, -38.33), (4, -524.76508) $-2.8(3.7)^{x}$

40. Solve the following system of equations (round to the nearest hundredth).

$$\begin{aligned} 3.2x - 5.7y &= 4 \\ -2.4x + 3.8y &= 7 \end{aligned}$$

(-36.25, -21.05)

41. Solve the following system of equations (write rational solutions in fractional notation, and round irrational solutions to the nearest tenth).

$$\begin{aligned} 2x - 3y + z &= 5 \\ 6y - 7z &= 4 \\ x - 4z &= 1 \end{aligned}$$

$$\boxed{\left(\frac{51}{11}, \frac{19}{11}, \frac{10}{11}\right)}$$

42. Solve the following system of equations (write rational solutions in fractional notation, and round irrational solutions to the nearest tenth).

$$\begin{aligned} 3x - y &= 0 \\ 2y + z &= 5 \\ 7x - z &= 1 \end{aligned}$$

$$\boxed{\left(\frac{6}{13}, \frac{18}{13}, \frac{29}{13}\right)}$$

43. Solve the quadratic inequality, and write your solution (if it exists) in interval notation (round to the nearest tenth if necessary).

$$-.5x^2 - 2.4x + 18.6 < 9.76$$

$$\boxed{(-\infty, -6.8) \cup (2.0, \infty)}$$

44. Solve the inequality, and write your solution (if it exists) in interval notation (round to the nearest tenth if necessary).

$$x^4 - 2x^3 - 38.23x^2 + 23.63x + 144.2376 \geq 0$$

$$\boxed{(-\infty, -5.2] \cup [-1.8, 2.3] \cup [6.7, \infty)}$$

45. Find all points of intersection of the graphs of the given equations (round to the nearest hundredth if necessary).

needs to be corrected

$$(x-4)^2 + (y+1)^2 = 256$$
$$(x-5)^2 - (y-2)^2 = 121$$

(-8.22, 9.33), (-10.17, -8.44), (17.41, 7.73), (18.98, -6.63)

46. Solve the following equation (if necessary, round to the nearest hundredth).

$$x^4 + 0.39x^3 - 4.6042x^2 - 0.37638x + 3.677814 = 0 \quad \boxed{x \in \{-2.10, -1.01, 1.02, 1.70\}}$$

47. Factor the following polynomial completely (use only integers – no fractions and no rounding).

(x + 12)(x + 3)(x + 1)(x – 2)(x – 4)(x – 5)

$x^6 + 5x^5 - 87x^4 + 43x^3 + 902x^2 - 672x - 1440$

Use the matrices defined below for the following four problems. Do not use decimal notation in your matrix entries. Use fraction notation for rational numbers (excluding integers).

$$A = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 0 & -4 \\ 1 & 1 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 3 & 2 \\ -4 & 1 & 6 \\ 2 & -3 & 8 \end{bmatrix}$$

48. Find AB.

$$AB = \begin{bmatrix} -6 & -17 & 34 \\ \cancel{8}\,7 & \cancel{24}\,21 & -\cancel{34}\,-26 \\ 15 & -17 & 64 \end{bmatrix}$$

← corrected

49. If $f(x) = 2x^4 + x^3 - 8x^2$, evaluate $f(B)$.

$$\begin{bmatrix} 3623 & -2301 & 7106 \\ 4028 & -509 & 678 \\ 5666 & 141 & 6362 \end{bmatrix}$$

50. Find the multiplicative inverse of matrix A (remember to use fractions – not decimals).

$$A^{-1} = \begin{bmatrix} -\frac{5}{24} & \frac{1}{8} & \frac{5}{24} \\ \frac{13}{12} & \frac{3}{4} & -\frac{1}{12} \\ -\frac{1}{8} & -\frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

← needs to be corrected

CRYPTOLOGY:

A cryptogram is a message written according to a secret code. Matrix multiplication can be used to encode and decode messages. First we assign a number to each letter of the alphabet (A = 1, B = 2, C = 3, ..., Z = 26) and assign the number 0 to a blank space. Then any message is converted to numbers and partitioned into uncoded row matrices, each with the appropriate number of entries.

For example, if our encoding matrix is a 3X3 matrix, then we would partition the message "MEET ME MONDAY" into the following 1X3 row matrices.

[13 5 5] [20 0 13] [5 0 13] [15 14 4] [1 25 0]

To encode the message we might use the matrix $A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & -4\\ -2 & -3 & 5\end{bmatrix}$ as follows

Uncoded Message	Encoding Matrix A	Coded Message

$$\begin{bmatrix}13 & 5 & 5\end{bmatrix}\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & -4\\ -2 & -3 & 5\end{bmatrix}=\begin{bmatrix}-8 & -10 & 31\end{bmatrix}$$

51. Given the encoding matrix $A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & -4\\ -2 & -3 & 5\end{bmatrix}$, encode the following message.

CAPTAIN OLIVER LOVES TO DRIVE

13, 19, 119, -28, 29, 138, -13, 29, 161, 25, 34, 157, 44, 5, -70, 43, 34, 127, 47, 5, -75, 5, 20, 5, 55, 13, -11, -29, 22, 63

52. Given that the encoding matrix $A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & -4\\ -2 & -3 & 5\end{bmatrix}$ was used to create the following coded message, decode the message.

13, 12, 228, 124, 19, 158, 29, -33, 98, -3, 52, 202, 135, -3, 198, 113, -12, 182, 50, 26, 94, 80, 48, 32

MONTANA LOVES TO SLEEP

53. Draw a skate-boarder moving across your graphing window. Show it to one of the instructors (bring your answer sheet with you).

END